Differentially Private Optimal Power Flow

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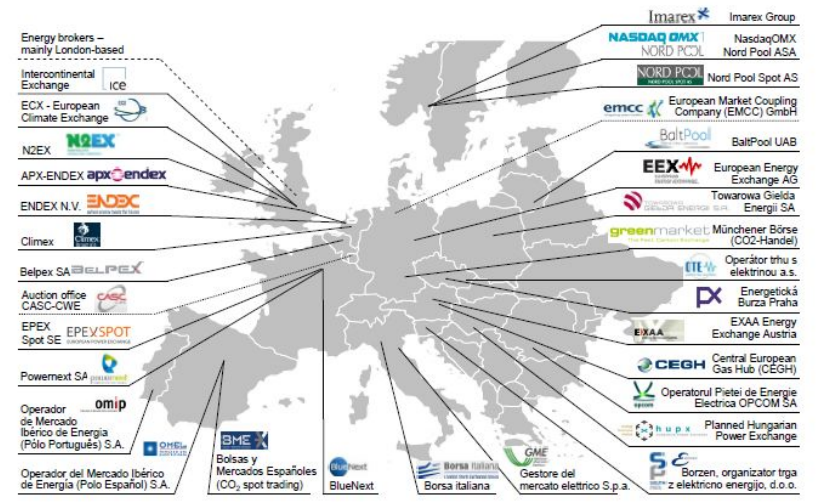
Power generation is decentralized

Continental scale



Complex markets, complex constraints

Physical

- Laws: Ohm's law, Kirchoff circuit laws
- Limits: Line capacity, flow rates, generation

Spatial

- Power sources in different regions
- Network structure of transmission lines

Temporal

- Time needed to ramp up/ramp down
- Respond to changing loads and demands

Optimal Power Flow: Background and Motivation

Coordination via optimization

Minimize the cost

- No two power plants are exactly alike (efficiency, cost, ...)

Optimize power delivery

- Set parameters: system voltages, “bus angles”, ...

Full problem: AC OPF

Faithful model of physics behind power network

- Objective: minimize total generation cost
- Constraints: voltage, current, and generation limits

A very thorny problem

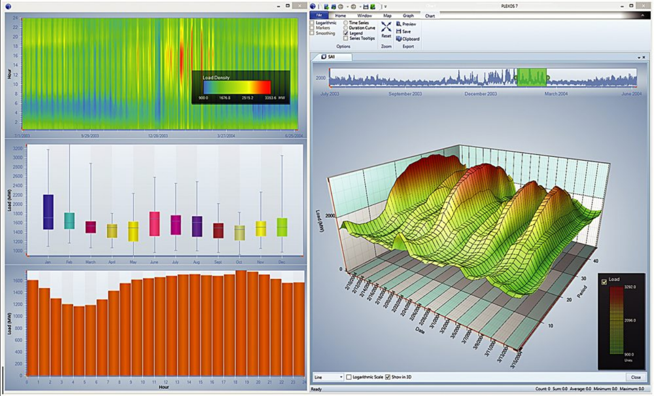
- Non-convex, discrete and continuous
- Complex valued: formulations involve sine, cosine, etc.

OPF problem is subject of a lot of research

Faster solutions enable faster grid response

- Convex SDP relaxations, investigate duality gap

Modeling tools for simulation and forecasting



Linearized version: DC OPF

LP version of AC OPF

- Simplify problem (remove current limits, voltages)
- Make small-angle approximations ($\sin(\theta) \approx \theta$)

Network model

- Generators located at nodes ("buses")
- Power can flow between neighboring nodes

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Generated power + net flow equals demand at each node

More formally

Constants

- Generation costs c_i, limits m_i, demands d_i (“loads”)
- Network matrix B (“bus susceptances”)

Variables

- Generation amount g_i and flow control θ_i (“bus angle”)

More formally: DC OPF linear program

$$\begin{aligned}\text{minimize: } & \sum_i c_i g_i \\ \text{subject to: } & g + B\theta = d \\ & -m \leq g \leq m \\ & -1/3 \leq \theta \leq 1/3\end{aligned}$$

What about privacy?

Nodes have data

- Loads: how much power is being demanded at each node?
- Costs: how much does power generation cost?

Solution naturally split between nodes

- Tell each power plant how much to generate
- Protect agent's data from joint view of other agents

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More formally, this calls for...

Joint Differential Privacy

Solving the DC OPF Problem under Joint Differential Privacy

Idea: net flow at each node is low sensitivity

Net flow: Flow out minus flow in

- Vector $B\theta$ gives the (signed) net flow at each node
- Potential: flow $\approx \theta_i - \theta_j$ for neighboring (i, j)

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What happens if one node's load changes by Δ?

- Generate more power at cheapest nodes with capacity
- Total absolute change in net flows at most 2Δ

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DC OPF constraints: conservation of flow

Compute generation P_g from noisy net flows

Three step process:

1. Solve original DC OPF problem exactly, get θ^*
2. Compute noisy flows:

$$\hat{f} = B\theta^* + \mathsf{Lap}_{\epsilon/2\Delta}$$

3. Compute generation:

$$g_i + \hat{f} = d_i$$

What about the bus angles θ?

OK to publish noisy flows, but not bus angles

- Need this info to induce noisy flow:

$$\hat{\theta} \text{ such that } \hat{f} = B\hat{\theta}$$

Possible problems

- Out of range: $\hat{\theta}$ too big/too small?
- No consistent solution for $\hat{\theta}$?

More refined: Projected Laplace mechanism

Add noise and project

- Polytope of noisy flows realizable by valid bus angles
- Add Laplace noise to get noisy net flows $\hat{f}$
- Project to polytope, solve for $\hat{\theta}$ and publish

Wrapping Up:
Three Takeaways

JDP is a good fit for graph problems

Data associated with each node

- Local pieces of solution can be distributed to nodes
- Handy link parts of inputs and parts of solutions

Relaxations of JDP possible

- Graph structure gives relation between agents
- Protect data against just the joint view of neighbors?

More to do for private optimization

Plenty of past work on private optimization

- Linear programs
- (Separable) convex programs

“Minor” assumptions are not always minor

- Know optimal value
- Only inequality (one-sided) constraints
- Can violate constraints “a little bit”

Many other uses for privacy in power flow problems

Further directions

- Protect more data (network structure?)
- Improve accuracy for specific network topologies
- Handle richer problems, towards AC OPF

Possible game theory and mechanism design angles

- Achieve approximate truthfulness
- Make it harder for agents to signal through costs
- Compute market clearing prices

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